Journal of Applied Mechanics and Technical Physics, Vol. 41, No. 2, 2000

UNSTEADY RADIATIVE-CONDUCTIVE HEAT TRANSFER IN A SEMITRANSPARENT SELECTIVELY ABSORBING LAYER

A. L. Burka and N. A. Rubtsov

UDC 536.244

Results of a numerical solution of the boundary-value problem of radiative-conductive heat transfer in a flat layer of a selectively absorbing and radiating medium are presented. The effect of the optical properties of the medium and the walls, the temperature of the source of radiation, and the relationship between the absorption spectra of the medium and the source of radiation on temperature distribution is studied.

The extensive use of semitransparent materials selectively absorbing thermal radiation in various fields of science and technology is associated with the high requirements of their production technology. Much attention is paid to calculations of unsteady radiative-conductive heat transfer (RCHT) in semitransparent media because of the necessity of mathematical modeling of the processes of complex heat transfer in these media at high temperatures. In this aspect, typical processes are the heating of glass billets in technologies of their melting, casting, and moulding.

A series of papers [1–5] is devoted to development of calculation methods for studying unsteady RCHT in semitransparent media. Primary attention there is paid to studying the effect of the optical properties of semitransparent materials on the formation of the temperature field. It should be noted that, apart from the optical properties of a semitransparent material, a significant effect on heat transfer at high temperatures is exerted by the selective character of the source of radiation used for heating. This problem is studied in few papers. Chel'tsova and Shakhmatova [6] considered the model of a complex source whose radiation is characterized by different temperatures and spectral emissivities. By averaging the balance equation over the layer thickness, the problem of determination of the temperature profile was reduced to the solution of a system of time-dependent nonlinear ordinary differential equations for the mean temperature of each sublayer into which the main semitransparent layer was divided. Kantorovich [7] studied RCHT in a layer of a selectively absorbing and radiating medium with account of the spectral character of the radiation flux incident onto the edge of the layer.

In the present work, we study the process of radiative heating of a layer of a semitransparent selectively absorbing and radiating medium as applied to heating of an LK-7 window-glass plate. In contrast to [6] where the boundary-value problem for the balance equation was solved in the approximation of temperature averaging over the layer, the unsteady boundary-value problem for the heat-conduction equation is solved here in a rigorous statement.

The mathematical formulation and the method for solving the system of energy-balance and radiation-transport equations are given. The energy equation and the boundary conditions have the following form:

$$\rho c \frac{\partial T}{\partial t} = \Lambda \frac{\partial^2 T}{\partial x^2} - \frac{\partial E}{\partial x}, \qquad 0 < x < L, \qquad t > 0; \tag{1}$$

Kutateladze Institute of Thermal Physics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 41, No. 2, pp. 124–129, March–April, 2000. Original article submitted June 25, 1998; revision submitted September 7, 1998.

0021-8944/00/4102-0319 $25.00

$$\Lambda \frac{\partial T}{\partial x} = \alpha_1(T - T_1) - \int\limits_{\Omega_1} \varepsilon_{1\nu}[Q_{1\nu}(T_1^*) - E_{1\nu}(T)]\, d\nu, \qquad x = 0; \tag{2}$$

$$\Lambda \frac{\partial T}{\partial x} = \alpha_2(T_2 - T) + \int\limits_{\Omega_2} \varepsilon_{2\nu}[Q_{2\nu}(T_2^*) - E_{2\nu}(T)]\, d\nu, \qquad x = L; \tag{3}$$

$$T(x, 0) = T_0(x). \tag{4}$$

The equations of radiation-energy transport with boundary conditions are written as

$$\mu \frac{dI_\nu^+}{dx} + æ_\nu I_{p\nu}^+ = æ_\nu I_{p\nu}(T), \qquad 0 < x < L; \tag{5}$$

$$\mu \frac{dI_\nu^-}{dx} - æ_\nu I_{p\nu}^- = æ_\nu I_{p\nu}(T), \qquad 0 < x < L; \tag{6}$$

$$I_\nu^+(0, \mu) = (1 - R_{1\nu}) I_{p\nu}(T_1^*) + R_{1\nu} I_\nu^-(0, \mu); \tag{7}$$

$$I_\nu^-(L, \mu) = \begin{cases} I_\nu^+(L, \mu), & 0 \leqslant \mu \leqslant \mu_\Gamma, \\ (1 - R_{2\nu}) I_{p\nu}(T_2^*) + 2R_{2\nu} \int\limits_{\mu_\Gamma}^{1} I^+(L, \mu)\mu\, d\mu, & \mu_\Gamma \leqslant \mu \leqslant 1. \end{cases} \tag{8}$$

Here $I_{p\nu}(T) = \dfrac{2\pi n^2 h\nu^3}{c_0^2[\exp(h\nu/(kT)) - 1]}$, $E_{i\nu}(T_i^*) = I_{p\nu}(T_i^*)$, $\dfrac{\partial E}{\partial x} = \int\limits_0^\infty æ_\nu[4I_{p\nu}(T) - G_\nu(x)]\, d\nu$, $G_\nu(x) = 2\pi \int\limits_0^1 [I_\nu^+(x, \mu) + I_\nu^-(x, \mu)]\, d\mu$, $\mu = |\cos\varphi|$, $\mu_\Gamma = \sqrt{1 - 1/n^2}$, $\mu_\Gamma = \cos\varphi_\Gamma$, φ_Γ is the angle of total internal reflection, φ is the angle between the ray and the positive direction of the x axis, $æ_\nu$ is the volumetric absorptivity of the material for the frequency ν, n is the refractive index, c is the heat capacity, ρ is the density of the medium, Λ is the thermal conductivity, L is the layer thickness, T_i and T_i^* are the temperatures of the ambient medium and external radiators, $I_\nu^\pm$ are the spectral intensities of radiation in the positive and negative directions of the x axis, $I_{p\nu}$ is the Planck function, $Q_{i\nu}$, $E_{i\nu}$, $\varepsilon_{i\nu}$, $R_{i\nu}$, and Ω_i are the densities of the incident fluxes, intrinsic radiation, emissivity, reflectivity, and spectral regions of opacity of the boundary surfaces, respectively, and α_i are the coefficients of convective heat transfer from the plate at the boundaries $(i = 1, 2)$.

From the energy and radiation-transport equations with the corresponding boundary conditions, it follows that the temperature distribution in the glass layer is determined to a large extent not only by the selective character of the external source of radiation but also by the optical properties of the surfaces of the semitransparent layer. In particular, condition (8) indicated that refraction and complete internal reflection of radiation are observed in the region of semitransparency of the material at the boundary $x = L$. It follows from (7) that the boundary $x = 0$ diffusely transmits and reflects selective radiation. Assuming $\alpha_1 = 0$, $\alpha_2 = 0$, $\varepsilon_{1\nu} = 0$, and $Q_{2\nu}(T_2^*) = I_{p\nu}(T_2^*)$ in Eqs. (2) and (3) and $R_{1\nu} = 1$ in (7), the boundary conditions for the temperature and intensity are written as [6]

$$\Lambda \frac{\partial T}{\partial x} = 0, \qquad x = 0; \tag{9}$$

$$\Lambda \frac{\partial T}{\partial x} = \int\limits_{\Omega_2} \varepsilon_{2\nu}[I_{p\nu}(T_2^*) - I_{p\nu}(T)]\, d\nu, \qquad x = L; \tag{10}$$

$$I_\nu^+(x, \mu) = I_\nu^-(x, \mu), \qquad x = 0, \tag{11}$$

where $\varepsilon_{2\nu} = 1/(1/\varepsilon_{s,\nu} + 1/\varepsilon_{g\nu})$ and $\varepsilon_{s,\nu}$ and $\varepsilon_{g,\nu}$ are the emissivities of the external source of radiation and glass in the opacity region.

Using the Green function, boundary-value problem (1)–(4) with account of (9)–(11) reduces to the nonlinear integral equation in the dimensionless temperature

$$\theta(\xi, t) = q(\theta)\Gamma(\xi, 1) + \int_0^1 F(\theta, z, t)\Gamma(\xi, z)\, dz, \tag{12}$$

where

$$q(\theta) = \frac{L}{\Lambda T_2^*} \int_{\Omega_2} \varepsilon_{2\nu}[I_{p\nu}(\theta) - I_{p\nu}(T_2^*)]\, d\nu \quad (\xi = 1); \quad F(\theta, \xi, t) = \frac{G_R(\xi)}{\Lambda} + \frac{c\rho L^2}{\Lambda} \frac{\partial\theta}{\partial t};$$

$$G_R(\xi) = L \int_0^\infty æ_\nu [4I_{p\nu}(\theta) - G(\xi)]\, d\nu, \quad \xi = x/L, \quad \theta(\xi, t) = T(\xi, t)/T_2^*.$$

The Green function has the form

$$\Gamma(\xi, z) = \begin{cases} -\cosh\xi \cosh(1 - z)/\sinh 1, & \xi \leqslant z, \\ -\cosh(1 - \xi)\cosh z/\sinh 1, & \xi \geqslant z. \end{cases}$$

The divergence of the radiation flux $dE/d\xi$ is expressed in terms of radiation intensities I_ν^+ and I_ν^-, which are determined from the solution of boundary-value problem (5)–(8) for the transport equation with account of (11) and have the form

$$I_\nu^+(\xi, \mu) = \left\{ I_\nu^+(0, \mu) + \frac{\tau_\nu}{\mu} \int_0^\xi I_{p\nu}(y) \exp\left(\frac{\tau_\nu}{\mu} y\right) dy \right\} \exp\left(-\frac{\tau_\nu}{\mu}\xi\right), \tag{13}$$

$$I_\nu^-(\xi, \mu) = \left\{ I_\nu^-(1, \mu) + \frac{\tau_\nu}{\mu} \int_\xi^1 I_{p\nu}(y) \exp\left[\frac{\tau_\nu}{\mu}(1 - y)\right] dy \right\} \exp\left[-\frac{\tau_\nu}{\mu}(1 - \xi)\right], \tag{14}$$

where $\tau_\nu = æ_\nu L$. Explicit relations for the boundary intensities are obtained by solving a system of two algebraic equations in $I_\nu^+(0, \mu)$ and $I_\nu^-(1, \mu)$ using relations (8), (11), (13), and (14). Substituting the values of $I_\nu^+(0, \mu)$ and $I_\nu^-(1, \mu)$ into (13) and (14), we obtain the final relations for the intensities entering into the radiation-flux divergence $dE/d\xi$.

Thus, the RCHT problem (1)–(8) in a flat layer of a selectively absorbing and radiating medium reduces to the solution of the nonlinear integral equation (12) relative to temperature by the Newton–Kantorovich iteration method [8]. The integrals in (12)–(14) were calculated by Gauss' quadrature formulas with 20 nodes, and the derivative $\partial\theta/\partial t$ was approximated by a finite-difference relation. The temperature profile was calculated for each moment of time.

Numerical calculations with account of the selective character of radiation were conducted for an LK-7 glass with the optical and thermophysical characteristics $\Lambda = 1.42$ W/(m · K), $c = 1.17 \cdot 10^3$ J/(kg · K), $\rho = 2.44 \cdot 10^3$ kg/m^3, $n = 1.5$, and $L = 0.01$ m [6]. According to [6], the absorption spectrum of glass is characterized by four bands located in the following wavelength ranges λ [μm]: (0, 1.3), (1.3, 2.5), (2.5, 5), and (5, ∞). The values of the absorption factors $æ_\lambda$ [m^{-1}] for these wavelength ranges are 0.2, 10, 320, and ∞. In the region of opacity (5, ∞), the emissivity of glass is $\varepsilon_g = 1$.

In accordance with the identified characteristic absorption bands of glass, the emissivity of the source $\varepsilon_{s,\lambda}$ is divided into four steps of unit or zero height (1, 0). Thus, for $\varepsilon_{(0,1.3)} = 1$, $\varepsilon_{(1.3,2.5)} = 1$, $\varepsilon_{(2.5,5)} = 0$, and $\varepsilon_{(5,\infty)} = 0$ for the source temperature $T_s = 3000$ K, we assume that $\varepsilon_{s,\lambda} = (1, 1, 0, 0)$.

Results of numerical simulation of the process of heating of a glass plate by radiation are presented below. Figure 1a shows the temperature on the plate surface ($x = 0$), which is not irradiated from outside (or which is thermally insulated), versus time for the temperature of the source of radiation $T_s = 3000$, 2100, and 1500 K and different distributions of emissivity $\varepsilon_{s,\lambda}$ over the radiation absorption bands in glass. The

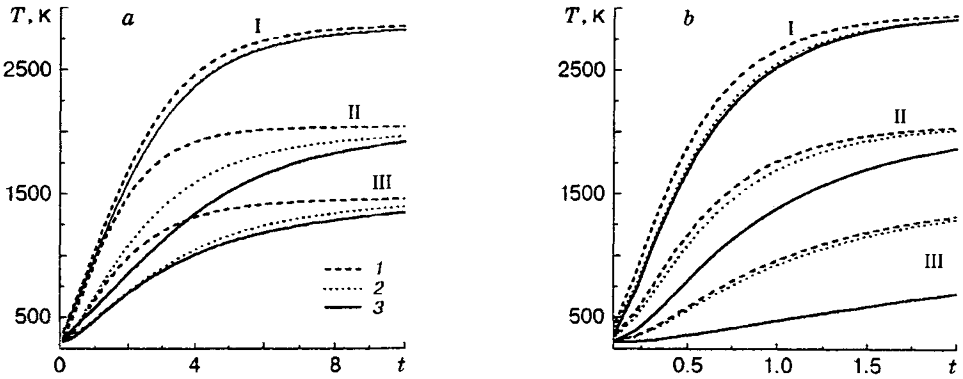

Fig. 1. Temperature of a thermally insulated surface ($x = 0$) versus time for opacity regions $5\ \mu m \leqslant \lambda < \infty$ (a) and $0 \leqslant \lambda < \infty$ (b): for region I ($T_S = 3000$ K), curves 1, 2, and 3 refer to $\varepsilon_{S,\lambda} = (1, 1, 0, 0)$, (1, 0, 0, 0), and (0, 1, 0, 0), respectively, for region II ($T_S = 2100$ K), curves 1, 2, and 3 refer to $\varepsilon_{S,\lambda} = (0, 1, 1, 0)$, (0, 1, 0, 0), and (0, 0, 1, 0), respectively, and for region III ($T_S = 1500$ K), curves 1, 2, and 3 refer to $\varepsilon_{S,\lambda} = (0, 0, 1, 1)$, (0, 0, 1, 0), and (0, 0, 0, 1), respectively.

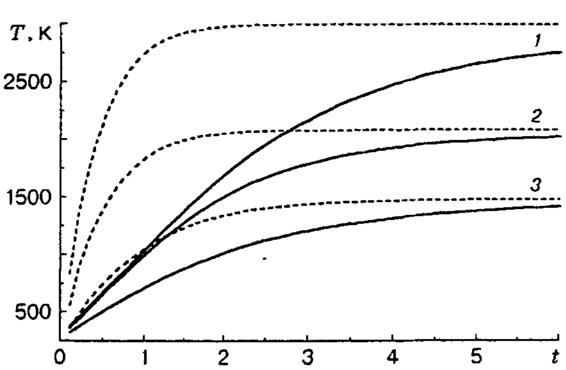

Fig. 2

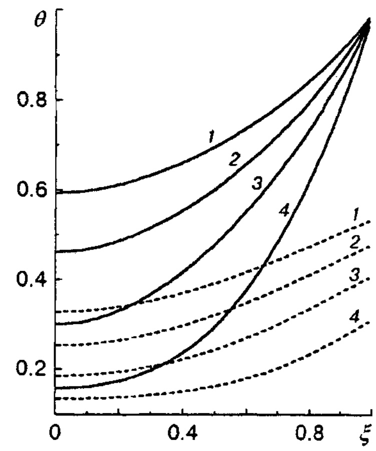

Fig. 3

Fig. 2. Temperature on the plate surface $x = L$ versus time for $0 \leqslant \lambda \leqslant 5\ \mu m$ (dashed curves) and $5\ \mu m \leqslant \lambda < \infty$ (solid curves); curves 1 refer to $T_S = 3000$ K and $\varepsilon_{S,\lambda} = (1, 1, 0, 0)$, 2 to $T_S = 2100$ K and $\varepsilon_{S,\lambda} = (0, 1, 1, 0)$, and 3 to $T_S = 1500$ K and $\varepsilon_{S,\lambda} = (0, 0, 1, 1)$.

Fig. 3. Distribution of the temperature θ over the thickness of a plate ($0 \leqslant \xi \leqslant 1$) irradiated by a source with $T_S = 3000$ K and $\varepsilon_{S,\lambda} = (1, 1, 1, 1)$ for $5\ \mu m \leqslant \lambda < \infty$ (dashed curves) and $0 \leqslant \lambda < \infty$ (solid curves); curves 1, 2, 3, and 4 refer to $t = 0.33$, 0.22, 0.11, and 0.01 h, respectively.

shift of the maximum value of emissivity $\varepsilon_{S,\lambda} = 1$ to the region of short wavelengths significantly increases the temperature, increases the rate of temperature variation, and reduces the time needed for the heated system to reach a steady regime in the case $T_S = 3000$ K as compared to the case of a moderate temperature of the source of radiation $T_S = 1500$ K. We took into account the division of the overall range of the absorption factor $æ_\lambda$ into four characteristic bands including the opacity region $(5, \infty)$. The presence of this region in the absorption spectrum of glass indicates that the radiation is completely absorbed by the surface layer $x = L$ within the wavelength range from 5 μm to infinity, and then it is transferred by means of RCHT to the deep layers to the surface $x = 0$.

Figure 1b shows the temperature dependences of a thermally insulated plate surface on time, which refer to a hypothetical case of existence of a highly heat-conducting opaque sublayer with the surface-opacity

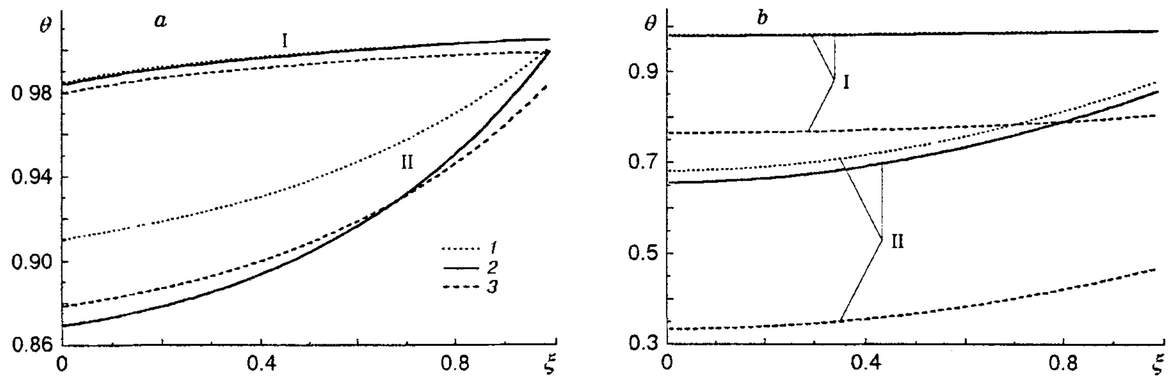

Fig. 4. Distribution of the temperature θ over the plate thickness ($0 \leqslant \xi \leqslant 1$) for $t = 1.11$ (a) and 5.55 (b) h; for region I ($T_S = 3000$ K), curves 1 refer to $\varepsilon_{s,\lambda} = (1, 1, 0, 0)$, 2 to $\varepsilon_{s,\lambda} = (1, 0, 0, 0)$, and 3 to $\varepsilon_{s,\lambda} = (0, 1, 0, 0)$; for region II ($T_S = 1500$ K), curves 1 refer to $\varepsilon_{s,\lambda} = (0, 0, 1, 1)$, 2 to $\varepsilon_{s,\lambda} = (0, 0, 1, 0)$, and 3 to $\varepsilon_{s,\lambda} = (0, 0, 0, 1)$.

region $0 \leqslant \lambda < \infty$ on the irradiated surface of the glass plate ($x = L$). A comparison of Fig. 1a and b shows that the glass temperature and heating rate increase significantly in this case. The assumption of opacity of the irradiated surface is used in calculations whose results are plotted in Fig. 2 as the temperature of the irradiated surface of the glass plate versus time. Two opacity regions of the irradiated glass surface are considered: 5 μm $\leqslant \lambda < \infty$ (solid curves) and $0 \leqslant \lambda \leqslant 5$ μm (dashed curves). Obviously, the second case is only hypothetical. It follows from Fig. 2 that the presence of an opacity region within the wavelength range from 0 to 5 μm on the irradiated surface of the plate leads to significant intensification of the process of heating of the glass plate as compared to heating of glass with an opacity region within the range from 5 μm to ∞.

The calculation results for dimensionless temperature fields on the plate are plotted in Figs. 3 and 4. Figure 3 shows the kinetics of plate heating at initial times by a source of radiation with $T_S = 3000$ K and $\varepsilon_{s,\lambda} = (1, 1, 1, 1)$. The processes of unsteady RCHT in a glass plate with an opacity region 5 μm $\leqslant \lambda < \infty$ under conditions of weak nonisothermality are proceeding slowly. The presence of a heating black surface $x = L$ of the glass layer considerably intensifies the heating process. Clearly expressed nonisothermality over the layer thickness is observed, which can favor the formation of convective flows in the glass melt.

Figure 4 shows the distributions of the dimensionless temperature over ξ in the plate with an opaque irradiated surface $x = L$ (with an opacity region $0 \leqslant \lambda < \infty$) at the times $t = 1.11$ h and $t = 5.55$ h when the system reaches a regime close to the steady state. The temperature distributions have a quasi-isothermal character and differ weakly for all kinds of radiation sources and their spectral emissivities $\varepsilon_{s,\lambda}$. An exception is the case of a source of radiation with $T_S = 1500$ K and $\varepsilon_{s,\lambda} = (0, 0, 0, 1)$ coinciding with the fourth absorption band from 5 μm to infinity for which the absorption factor in glass is infinitely great. In this case, obviously, the pertinent factor in the process of thermal-energy transfer is the thermal conductivity. The process of reaching a steady temperature regime is rather extended in time. The quasi-isothermal character of temperature distribution is determined by the condition of the problem related to thermal insulation of the nonirradiated surface of the plate ($x = 0$).

The character of distribution of the dimensionless temperature over the plate thickness at some intermediate time t depends significantly on the temperature of the source of radiation and on the distribution of spectral emissivities $\varepsilon_{s,\lambda}$ over the absorption bands. The degree of influence of the latter is determined by the character of displacement of radiation maxima along the wavelength scale in accordance with Wien's law.

The present study of unsteady radiative-conductive heat transfer in a glass plate heated by a model source of radiation indicates that the processes of thermal-energy transfer in glass are rather complex and depend on the optical properties of radiation sources, glass material, and plate boundaries. The results obtained can be used to improve the technology of heating glasses and melts.

REFERENCES

1. A. L. Burka, "Taking into account the temperature dependence of the absorption factor in studying complex heat transfer," in: *Heat Transfer by Radiation* (collected scientific papers) [in Russian], Inst. of Thermal Physics, Novosibirsk (1977), pp. 24–31.
2. A. L. Burka, N. A. Rubtsov, and V. P. Stupin, "Theoretical and experimental study of heating regimes for Plexiglas," in: *Heat and Mass Transfer-VI*, Materials of VI All-Union Conf. [in Russian] Vol. 2, Minsk (1980), pp. 132–137.
3. N. V. Marchenko, B. I. Aronov, and Ya. I. Shtipel'man, "Calculation of unsteady radiative-conductive heat transfer in a flat layer of a selective medium with semitransparent boundaries," *Teplofiz. Vys. Temp.*, **18**, No. 5, 1007–1017 (1980).
4. I. P. Makarova, V. D. Chel'tsova, and I. P. Shakhmatova, "Calculation of unsteady radiative-convective heat transfer in a layer of variable thickness," *Teplofiz. Vys. Temp.*, **22**, No. 1, 99–104 (1984).
5. A. L. Burka, "Unsteady combined heat transfer with account of scattering anisotropy," *Teplofiz. Vys. Temp.*, **25**, No. 1, 110–115 (1987).
6. V. D. Chel'tsova and I. P. Shakhmatova, "Investigation of the process of heating of a semitransparent layer by a selective source of radiation," *Teplofiz. Vys. Temp.*, **23**, No. 6, 1135–1141 (1985).
7. A. L. Burka, "Transient radiative-conductive heat transfer in a flat layer of a selectively absorbing and radiating medium," *Prikl. Mekh. Tekh. Fiz.*, **39**, No. 1, 105–109 (1998).
8. L. V. Kantorovich, "Newton's method," *Tr. Mat. Inst. Akad. Nauk SSSR*, **28**, 135–139 (1949).